Optimal Room Orientation to Minimize Solar Incidence

Manu Anish

August 2023

Contents

1 Introduction

1.1 Rationale

I live in Ahmedabad, a sunbaked tropical region in the far west of India. As one of the hottest regions in the country [\(Kalsi and Pareek](#page-18-1) p. 1), the scorching heat can be unbearable, surpassing 45 degrees Celsius (113 degrees Fahrenheit) during the peak summer months.

Figure 1 — The extreme temperatures from the heat wave of 2010. Image source: [\(Kakkad, Khyati, et al.](#page-18-2) p. 4)

These extreme temperatures have dire consequences, as evidenced by the devastating heat waves that struck Ahmedabad in 2010, resulting in over 100 deaths [\(Azhar et al.](#page-18-3) p. 4).

Furthermore, the relentless sunlight that permeates through our windows increasing the heat inside our homes. It not only raises the indoor temperature but also our the reliance on energy-intensive air conditioning systems, which further strain the power grid and contribute to environmental degradation.

In light of these issues, I am determined to explore and challenge my mathematical knowledge to address the heat problem. By investigating the optimal room orientation, I aim to find a viable solution that can reduce solar heat gain while maintaining a comfortable indoor environment.

2 Methodology

Figure $2 - A$ photo of my living room.

In this document I will specifically look at my living room and consider the orientations that it could be rotated to minimize the amount of sunlight that enters during the day.

2.1 The Model

Figure 3 — Rays of sunlight casted through the patio door. (Isometric)

For the ease of calculation, I will consider only the patio door and since it is the major source of sunlight in my living room.

Remark: Since the sun is negligibly distant from my house, I consider the sun a *directional light*; that is, parallel vectors.

2.2 Objective

My living currently room is oriented 20◦ North-West. I intend to consider different angles of orientation about $\mathcal O$ and find an "optimium" angle of orientation that minimizes the total sunlight entering my living room during the summer.

2.3 What is an "optimium" orientation?

For calculating the sunlight area the following factors are to be considered:

- My latitude $23.0225°N$ (constant)
- Hour of the day (variable)
- Day of the year (variable)
- Door dimensions $4ft \times 6ft$ (constant)
- Room Length 10ft (constant)
- Room Orientation (variable)

By an optimal orientation I refer to one for which the total sunlight area entering my room is minimal for all 24 hours of day and 365 days of the year.

3 Modeling the Solar Vector

All of the equations in this section are cited from Wikipedia and are not my original derivations. They are restated and mildly modified to fit my circumstance.

3.1 Equitorial Coordinates

By convention, we use the equitorial coordinate system [\(Equatorial coor](#page-18-4)[dinate system](#page-18-4) , Wikipedia) to specify positions of celestial objects. The basic setup is given below:

Figure 4 — Position of the sun. (Isometric)

The angle β denotes the **solar azimuth angle** [\(Solar azimuth angle](#page-19-3), Wikipedia). The angle α denotes the **solar altitude**, the complement of the [\(Solar zenith](#page-19-4) [angle](#page-19-4) , Wikipedia).

Remark: North (N) is given by $0°$ Azimuth [\(Azimuth](#page-18-5), Wikipedia), so β is measured clockwise relative to **N** and \mathcal{O} . Similarly, α is measured clockwise relative to the **Zenith** [\(Zenith](#page-19-5), Wikipedia) (z) and \mathcal{O} .

To calculate these angles we can use the equations cited from [\(Solar zenith angle](#page-19-4) , Wikipedia) and [\(Solar azimuth angle](#page-19-3) , Wikipedia). That is:

3.1.1 Solar Altitude α

Theorem 3.1: (Solar Altitude) The Solar Altitude α [\(Solar zenith](#page-19-4) [angle\)](#page-19-4) is given by:

 $\alpha = \sin^{-1}(\sin \Phi \sin \delta + \cos \Phi \cos \delta \cos h)$

Where:

- Φ is the observer's latitude.
- δ is the **declination** of the sun.
- \bullet h is the **hour angle** in local solar time.

3.1.1.1 Solar Zenith θ

Remark: (Solar Zenith) The Solar Zenith θ [\(Solar zenith angle\)](#page-19-4) is the complement of the solar altitude. Thus it is given simply by:

$$
\theta = 90^{\circ} - \alpha
$$

3.1.2 Solar Azimuth β

Theorem 3.2: (Solar Azimuth) The Solar Azimuth β [\(Solar azimuth](#page-19-3) [angle\)](#page-19-3) is given by:

$$
\sin \beta = \frac{-\sin h \cos \delta}{\sin \theta}
$$

Where:

- \bullet h is the **hour angle** in local solar time.
- δ is the **declination** of the sun.
- θ is the Solar Zenith.

Since my latitude Φ is going to be a constant, I have to define two more variables in order to compute the solar vector. That is, h the hour angle and δ the declination of the sun.

3.1.3 Declination δ

Theorem 3.3: (Declination of the Sun) The declination of the sun is given by:

$$
\delta = -23.44^{\circ} \cdot \cos \left[\frac{360^{\circ}}{365} \cdot (N+10) \right]
$$

Where N is the day of the year starting from $N = 0$.

3.1.4 Hour Angle h

Theorem 3.4: (Hour Angle) The hour angle in local solar time is given by:

$$
h = 15^{\circ} (LST - 12)
$$

Where LST is the hour of the day (including fractional minutes).

3.2 Creating the Model

Using Python [\(Python 3\)](#page-18-6), I converted the theorems above into functions. Using the library Matplotlib [\(Matplotlib\)](#page-18-7) I was able to create a 3D vector pointing towards the sun given my latitude, time of day, and day of the year.

Figure 5 — An interactive demo of the solar vector I created.

3.2.1 Path of the Sun

I created the demo mostly for intuition and to understand the behavior of the Sun throughout the year. Messing around with it for a bit, I quickly realized that the sun generally rises from the East and sets towards the West. This is a crucial observation, as I can intuitively guess a minimal orientation would be at either 90◦ or 270◦ as this allows the patio door to be parallel to the Sun's movement. The room is therefore largely affected by overhead sunlight which is minimal as opposed to the directly lateral sunlight at $0°$ and $180°$.

4 Calculating Solar Incidence

Figure 6 — Calculating sunlight area with the solar vector.

Suppose the given solar vector is \vec{v} with azimuth β and altitude α . Notice that all rays will be parallel since we assume the sun is a directional light.

It follows that $AC \parallel ON$. Then we have ∠ACO is α , and since AB $\parallel MN$ we find that $\triangle ABC \sim \triangle MNO$. Then, $\angle C = \alpha$ and $AB = 6$ ft. Therefore, $tan(\alpha) = \frac{6 \text{ft}}{BC}$. Rearranging, we can solve for *BC*:

$$
BC = \frac{6 \text{ft}}{\tan(\alpha)}
$$

Another observation is that $ED \parallel BC$ and since $FA \parallel EB \implies EB \parallel CD$. In other words, $\Box B CDE$ is a rhombus.

4.1 Finding the area of $\Box B CDE$

We know that $\angle IOM = \beta + \gamma$ where γ is the orientation of my living room. Then since ∠HOL is an opposite angle it follows ∠IOM = ∠HOL. Then since OL \parallel BC we have $\angle HBC = \angle HOL = \beta + \gamma$. Here is a top-down view for reference:

Figure 7 — Top down view of the sunlight.

The area of the sunlight is going to be BASE \times HEIGHT, since we know $\Box B CDE$ is a rhombus. We know the base $BE = 4$ ft so we just need to find the height CC' . Since, $sin(B) = \frac{CC'}{BC}$ we find:

$$
CC' = \sin(B) \cdot BC
$$

$$
CC' = \frac{\sin(\beta + \gamma) \cdot 6 \text{ft}}{\tan(\alpha)}
$$

Then the area of the sunlight is given by:

$$
[\Box B CDE] = CC' \times BC = \frac{\sin(\beta + \gamma) \cdot 6\text{ft}}{\tan(\alpha)} \times 4\text{ft} = \frac{\sin(\beta + \gamma) \cdot 24\text{ft}}{\tan(\alpha)}
$$

4.2 Corrections

While the derived equation for sunlight area is largely accurate, there are a few edge cases to consider.

4.2.1 Sign Corrections

Notice I care about the area of sunlight "inside" my living room. In other words, I can use the sign of $\frac{\sin(\beta+\gamma)}{\tan(\alpha)}$ and define the area as a piecewise function:

$$
[\Box B CDE] = \begin{cases} \frac{\sin(\beta + \gamma) \cdot 24\text{ft}}{\tan(\alpha)} & \text{if } \frac{\sin(\beta + \gamma)}{\tan(\alpha)} > 0\\ 0 & \text{if } \frac{\sin(\beta + \gamma)}{\tan(\alpha)} < 0 \end{cases}
$$

Therefore, when the area is negative (on the opposite side) I set the area to be 0 so that it is not including in my calculations.

4.2.2 Area Bounds

Observe for smaller values of $tan(\alpha)$ the sunlight area can exceed the size of my living room, thus skewing actual sunlight area to be much larger than it should be. (e.g when the sun is just a little above the horizon)

To deal with this, I bound the area of the sunlight to the length of my living room. As such, a maximum area of $4 \cdot 10 = 40$ ft. Then our finalized area function looks like:

$$
[\Box B CDE] = \begin{cases} \min\left(\frac{\sin(\beta + \gamma) \cdot 24\text{ft}}{\tan(\alpha)}, 40\text{ft}\right) & \text{if } \frac{\sin(\beta + \gamma)}{\tan(\alpha)} > 0\\ 0 & \text{if } \frac{\sin(\beta + \gamma)}{\tan(\alpha)} < 0 \end{cases}
$$

This ensures that the sun cannot produce unrealistically large areas.

4.3 Discrete Summation of Sunlight Area

For simplicity, I consider only discrete sums of integer hours and integer days. That is,

$$
\sum_{d=0}^{365} \left(\sum_{h=0}^{24} \left[\square BCDE \right] \right), \quad h, d \in \mathbb{Z}
$$

I then aim to minimize this value with varying values of γ . As such,

$$
\min\left(\{0 \le \gamma \le 360 \; : \; \sum_{d=0}^{365} \left(\sum_{h=0}^{24} \left[\Box BCDE \right] \right), \quad h, d \in \mathbb{Z}\}\right)
$$

5 Minimizing Solar Incidence

To find an optimal room orientation γ , I will graph the the orientation vs the discrete summation of sunlight area and use the argmin $[\Box B CDE]$ [\(Arg max\)](#page-18-8) to compute the minimal orientation:

5.1 Modeling Sunlight Area

I rewrote the discrete sums in python, and plotted the area against the orientation, here are the results:

Remark: The code is omitted here for readability, you can find the full source code in section [7.2](#page-22-0) of the appendix.

Figure 8 — Log plot of discrete sunlight area vs room orientation.

5.2 Optimal Room Orientation

To find the minima of $[\Box B CDE]$, I use the python library numpy specifically the numpy.argmin function. It uses an algorithm similar to Newton's Method to find local minima and compares it against existing ones to find a global minima. Remark: The code is omitted here for readability, you can find the full source code in section [7.3](#page-23-0) of the appendix.

Figure 9 — Optimal Room Orientation is 79 \textdegree (Log Plot)

6 Conclusion

So, using my knowledge of math I was able to find the optimal orientation of my room is 79° to minimize the amount of sunlight I have entering my room. While there are other factors that effect this value in practice, this gives us a rough estimate of how a room should be oriented to reduce solar incidence throughout the year.

6.1 Data Analysis (Making Sense of 79°)

79◦ aligns with my observations from section [3.2.1](#page-10-0) as 79◦ Azimuth is close to the East-West (90°, 270°) path. This can also been seen in the broad cyclicity observed in the graph every 180◦ . However, there is a slight distinction. Naturally we might ask why 73° is more optimal than say, $73^\circ + 180^\circ = 253^\circ$? This

difference can be attributed to varying declinations of the sun. Due to different latitudes, and the tilt of the Earth, the path of the sun is slightly skewed towards the North/South leading to an imbalance as seen in the graph.

6.2 Limitations of the Method

In reality the atmospheric conditions and diffusion of the light play a large role in determining the actual effect of heat from the sunlight entering the room [\(Diffuse reflection](#page-18-9) , Wikipedia). While the rhombic sunlight area provides a good approximation, in practice, we will likely have more scattered and reflected area of effect:

Figure 10 $-$ Diffused Light on a Surface. Image source [Diffuse reflection](#page-18-9)

Moreover, the dispersed light can contribute to localized heating on various surfaces, potentially increasing the overall heat gain.

6.3 Practical Applications

The idea of passive cooling is by no means novel. It is especially well-known as one of the most environmentally friendly means to combat climate change. For example, [\(Oropeza-Perez and Østergaard](#page-18-10) p. 2), considers a review of other such related passive cooling techniques.

The objective of this study is to provide findings that can complement and collaborate with existing research efforts. By doing so, reducing our reliance on environmentally detrimental active cooling systems.

Conventional cooling methods not only consume substantial amounts of energy but also contribute significantly to greenhouse gas emissions. Therefore, my results are intended to contribute to a broader strategy aimed at creating sustainable, environmentally conscious practices in the built environment.

6.4 Reflection

And there we have it! I thoroughly enjoyed this exploration, and the journey that brought me to the final result of 79◦ . Aligning this outcome with my initial intuition was particularly gratifying; this experience has unquestionably deepened my fascination with the subject.

The project is by no means complete. Sooner or later, I want to get back to this and enhance the accuracy of these results by accounting for the effects of light diffusion and atmospheric conditions. In the end, I hold the hope that my work will contribute to making the world a better, more sustainable place, one room at a time.

Works Cited

- Arg max. Wikipedia, June 2023, Page Version ID: 1161677835. [en.wikipedia.](http://en.wikipedia.org/w/index.php?title=Arg_max&oldid=1161677835#Arg_min) org/w/index.php?title=Arg [max&oldid=1161677835#Arg](http://en.wikipedia.org/w/index.php?title=Arg_max&oldid=1161677835#Arg_min) min. Accessed 4 Sept. 2023.
- Azhar, Gulrez Shah, et al. "Heat-Related Mortality in India: Excess All-Cause Mortality Associated with the 2010 Ahmedabad Heat Wave". PLoS ONE, vol. 9, no. 3, Mar. 2014, e91831. https://doi.org[/10 . 1371 / journal . pone .](https://doi.org/10.1371/journal.pone.0091831) [0091831.](https://doi.org/10.1371/journal.pone.0091831)
- Azimuth. Wikipedia, Aug. 2023, Page Version ID: 1170448401. [en.wikipedia.](http://en.wikipedia.org/w/index.php?title=Azimuth&oldid=1170448401) org / w / index. php ? title = Azimuth & oldid = 1170448401. Accessed 1 Sept. 2023.
- Diffuse reflection. Wikipedia, Nov. 2021, Page Version ID: 1055411516. [en .](http://en.wikipedia.org/w/index.php?title=Diffuse_reflection&oldid=1055411516) wikipedia.org/w/index.php?title=Diffuse_reflection & oldid= 1055411516 . Accessed 6 Sept. 2023.
- Equatorial coordinate system. Wikipedia, Apr. 2023, Page Version ID: 1149147383. [en.wikipedia.org/w/index.php?title=Equatorial](http://en.wikipedia.org/w/index.php?title=Equatorial_coordinate_system&oldid=1149147383) coordinate system&oldid= [1149147383.](http://en.wikipedia.org/w/index.php?title=Equatorial_coordinate_system&oldid=1149147383) Accessed 1 Sept. 2023.
- Kakkad, Khyati, et al. "Neonates in Ahmedabad, India, during the 2010 Heat Wave: A Climate Change Adaptation Study". Journal of Environmental and Public Health, vol. 2014, Mar. 2014, Publisher: Hindawi, e946875. https://doi[.org/](https://doi.org/10.1155/2014/946875)10. [1155/2014/946875.](https://doi.org/10.1155/2014/946875)
- Kalsi, S. R., and R. S. Pareek. "Hottest April of the 20th century over northwest and central India". Current Science, vol. 80, no. 7, 2001, Publisher: Temporary Publisher, pp. 867–73. JSTOR, [www.jstor.org/stable/24105739.](http://www.jstor.org/stable/24105739) Accessed 21 Aug. 2023.
- Matplotlib. [matplotlib.org/stable/index.html#.](http://matplotlib.org/stable/index.html) Accessed 2 Sept. 2023.
- Oropeza-Perez, Ivan, and Poul Alberg Østergaard. "Active and passive cooling methods for dwellings: A review". Renewable and Sustainable Energy Reviews, vol. 82, Feb. 2018, pp. 531–44. https://doi.org[/10.1016/j.rser.2017.](https://doi.org/10.1016/j.rser.2017.09.059) [09.059.](https://doi.org/10.1016/j.rser.2017.09.059)
- Python 3. Python.org, Aug. 2023. [www.python.org/.](http://www.python.org/) Accessed 2 Sept. 2023.
- Solar azimuth angle. Wikipedia, Aug. 2023, Page Version ID: 1170286659. [en.](http://en.wikipedia.org/w/index.php?title=Solar_azimuth_angle&oldid=1170286659) [wikipedia.org/w/index.php?title=Solar](http://en.wikipedia.org/w/index.php?title=Solar_azimuth_angle&oldid=1170286659)_azimuth_angle&oldid=1170286659. Accessed 1 Sept. 2023.
- Solar zenith angle. Wikipedia, June 2023, Page Version ID: 1159258763. [en.](http://en.wikipedia.org/w/index.php?title=Solar_zenith_angle&oldid=1159258763) [wikipedia.org/w/index.php?title=Solar](http://en.wikipedia.org/w/index.php?title=Solar_zenith_angle&oldid=1159258763)_zenith_angle&oldid=1159258763. Accessed 1 Sept. 2023.
- Zenith. Wikipedia, Aug. 2023, Page Version ID: 1171273127. [en.wikipedia.org/](http://en.wikipedia.org/w/index.php?title=Zenith&oldid=1171273127) [w/index.php?title=Zenith&oldid=1171273127.](http://en.wikipedia.org/w/index.php?title=Zenith&oldid=1171273127) Accessed 1 Sept. 2023.

7 Appendix

7.1 Interactive Solar Vector

Here is the source code for the interactive demo of the solar vector I made. The code uses two files library.py containing the mathematical functions to create the simulation and main.py using matplotlib to create the view.

7.1.1 library.py

```
1 import math
2
3 # Solar Altitude
4 def calculate_solar_altitude (latitude, declination,
        hour_angle ) :
\begin{array}{c|c|c|c|c} 5 & \texttt{rad\_latitude = math.radians (latitude)} \ \hline 6 & \texttt{rad\_declination = math.radians (decimal)} \end{array}6 rad_declination = math.radians (declination)
 \tau \parallel \qquad \texttt{rad\_hour\_angle} = \texttt{match} . \texttt{radians} (hour_angle)
8
9 solar_altitude = math . degrees (
10 math.asin(math.sin(rad_latitude) *
11 math.sin(rad_declination) +
_{12} | math.cos(rad_latitude) *
13 math.cos (rad_declination) *\begin{array}{c|c} \n & \text{math.} \ncos (rad\_hour\_angle )\n\end{array}15 )
16
17 return solar_altitude
18
19 # Solar Zenith
_{20} \vert def calculate_solar_zenith (solar_altitude):
_{21} solar_zenith = 90 - solar_altitude
22 return solar_zenith
23
\frac{24}{25} # Solar Azimuth<br>\frac{25}{25} def calculate s
   def calculate_solar_azimuth (hour_angle, declination,
        solar_zenith ) :
26 rad_hour_angle = math . radians ( hour_angle )
_{27} \mid rad_declination = math.radians (declination)
28 rad_solar_zenith = math.radians (solar_zenith)
29
```

```
30 | numerator = -math \, .sin(rad_hour\_angle) * math.cos (
           rad_declination )
31 denominator = math.sin(rad_solar_zenith)
32
33 solar_azimuth = math.degrees (math.asin (numerator /
            denominator ) )
34
35 return solar_azimuth
36
37 \# Declination of the Sun
\frac{38}{39} def calculate_declination (\frac{day\_of\_year}{day\_of\_year} +
39 angle = 360 / 365 * ( day_of_year + 10)
40 declination = -23.44 * math.cos(math.radians(angle))
41
42 return declination
43
44 \parallel # Hour Angle<br>45 \parallel def calculat
45 \vert \texttt{def} calculate_hour_angle (local_solar_time):
46 hour_angle = 15 \ast (local_solar_time - 12)
47 return hour_angle
```
7.1.2 main.py

```
\frac{1}{2} import numpy as np<br>import matplotlib.
 2 | import matplotlib . pyplot as plt
 _3 | from mpl_toolkits mplot3d import Axes3D
 4 from matplotlib .widgets import Slider
 5 from library import calculate_declination,
       calculate_hour_angle , calculate_solar_altitude ,
       calculate_solar_azimuth
6
7 \# Create a figure and a 3D axis
 8 \mid \text{fig} = \text{plt}. figure ()
 9 ax = fig.add_subplot(111, projection='3d')
10
11 # Set the initial values for day of the year, hour of the
       day , and latitude
12 |initial_day_of_year = 100
13 initial_hour_of_day = 12
_{14} |initial_latitude = 23.0225 \, # Default latitude
15
16 # Create sliders for day of the year, hour of the day, and
       latitude
17 | ax_day = plt.axes ([0.2, 0.01, 0.65, 0.03]) # Adjusted
      position
18 | ax_hour = plt. axes ([0.2, 0.05, 0.65, 0.03]) # Adjusted
      position
19 ax_latitude = plt. axes ([0.2, 0.09, 0.65, 0.03]) # Adjusted
      position
20
_{21} slider_day = Slider (ax_day, 'Day of the Year', 0, 365,
       valinit = initial_day_of_year )
_{22} slider_hour = Slider (ax\_hour, 'Hour of the Day', 0, 24,
       valinit = initial_hour_of_day )
_{23} \mid slider_latitude = Slider(ax_latitude, 'Latitude (degrees)',
       -90 , 90 , valinit = initial_latitude )
24
25 \# Function to update the plot based on slider values
26 def update (val):
27 day_of_year = slider_day.val
28 hour_of_day = slider_hour.val
_{29} | latitude = slider_latitude.val
30
31 # Calculate declination and hour angle
32 declination = calculate_declination (day_of_year)
33 hour_angle = calculate_hour_angle (hour_of_day)
```

```
34
35 # Calculate solar altitude and azimuth
\overline{\phantom{a}} solar_altitude = calculate_solar_altitude (latitude,
                   declination , hour_angle )
37 solar_azimuth = calculate_solar_azimuth (hour_angle,
                   declination, 90 - solar_altitude)
38
39 # Convert to radians for 3D vector calculation
_{40} solar_altitude_rad = np.radians(solar_altitude)
41 \vert solar_azimuth_rad = np. radians (solar_azimuth)
42
43 # Calculate the 3D vector components
\begin{array}{ccc} 44 & \times & = & \texttt{np.sin}(\texttt{solar\_azimuth\_rad}) & \ast & \texttt{np.cos}(\texttt{p. smin}(\texttt{m. smin}solar_altitude_rad )
45 y = np.cos (solar_azimuth_rad) * np.cos (
                  solar_altitude_rad )
46 \vert z = np.sin(solar_altitude_rad)
47
48 \parallel \parallel \parallel Clear the previous plot
49 ax. clear ()
50
\begin{array}{c|cccccc} 51 & & \text{\#} & \text{Add a } 1 \text{x} 1 \text{x} 1 & \text{plane at the origin } (0, 0, 0) \end{array}52 ax.plot_surface (
53 np.array ([[0, 1], [0, 1]]),
54 np.array ([[0, 0], [1, 1]]),
55 | p.array ([[0, 0], [0, 0]]),
56 color='gray',
\begin{array}{c|c}\n 57 & \text{alpha=0.5} \\
 58 & \text{alpha=0.5}\n \end{array}58 )
59
60 # Position the solar vector in the middle of the plane
\begin{array}{lllll} \circ\:\:\: & \quad \texttt{ax.quiver(0.5, 0.5, 0, x, y, z, color='red', label='} \end{array}Solar Vector', arrow_length_ratio=0.1)
62
63 \parallel \parallel # Add labels for North, South, East, and West
64 ax . text (0.5 , -0.05 , 0 , ' North ' , color = ' black ' ,
                    horizontalalignment = ' center ')
\begin{array}{cccc} \text{65} & \text{a} & \text{a} & \text{x} & \text{.text}(0.5, 1.05, 0, 3.001 \text{h} \text{)} \\ \text{56} & \text{b} & \text{b} & \text{c} & \text{d} \end{array}horizontalalignment = ' center ')
\begin{array}{ccc} \circ\circ & \quad\quad\quad \text{ax.text} \ (\texttt{-0.05, 0.5, 0, 'East'},\ \texttt{color='black',} \end{array}horizontalalignment = ' center ')
\begin{array}{c|c} \hline \text{67} & \text{72.1234444455} \end{array} ax.text(1.05, 0.5, 0, 'West', color='black',
                   horizontalalignment = 'center')
68
\begin{array}{c|c} \n\overline{69} & \text{ax.set\_xlim(0, 1)} \\
\hline\n\overline{70} & \text{ax.set\_ylim(0, 1)}\n\end{array}\begin{array}{cc} 70 \\ 71 \end{array} ax.set_ylim (0, 1)<br>\begin{array}{cc} 21 \text{ m} & 0 \\ 0 & 1 \end{array}\begin{array}{c} \n\text{71} \quad \text{ax.set\_zlim(0, 1)}\n\end{array}\begin{array}{cc} \mathbf{72} & \mathbf{a} \mathbf{x} \mathbf{.} \mathbf{set\_xlabel} \end{array}73 ax.set_ylabel('Y')
\begin{array}{c|c} \n\hline\n\text{74} & \text{ax.set_zlabel('Z')} \\
\hline\n\text{75} & \text{ax.length()}\n\end{array}ax.legend()
76
77 fig. canvas. draw ()
78
79 # Attach the slider update function to the slider events so slider_day.on_changed(update)
     slider_day.on_changed(update)
81 slider_hour.on_changed (update)
82 slider_latitude.on_changed (update)
83
\begin{array}{c|cc}\n 84 & # & \text{Initial plot} \\
 85 & \text{update}(\text{None})\n\end{array}update (None)
86
87 plt.show ()
```
7.2 Discrete Sums Plot

Here is the source code for plot of the discrete sums vs room orienation. The code refers to the old library.py in section [7.1.1](#page-19-2) of the appendix containing the mathematical functions for calculations and a new plot.py using matplotlib to create the view.

7.2.1 plot.py

```
1 import matplotlib.pyplot as plt
 2 import numpy as np
 3 from library import calculate_solar_altitude,
        calculate_solar_azimuth , calculate_declination ,
        calculate_hour_angle
 4
5 def calculate_sunlight_area (latitude, day_of_year,
        hour_of_day , room_orientation ) :
 6 declination = calculate_declination (day_of_year)
 \tau | hour_angle = calculate_hour_angle(hour_of_day)
 \begin{array}{|l|} \hline \begin{array}{l} 8 \end{array} \end{array} \end{array} \quad \begin{array}{l} \hline \begin{array}{l} \hline \begin{array}{l} \hline \begin{array}{l} \hline \begin{array}{l} \hline \begin{array}{l} \hline \end{array} \end{array} \end{array} \end{array}declination , hour_angle )
 9 solar_azimuth = calculate_solar_azimuth ( hour_angle ,
              declination , 90 - solar_altitude )
10
11 beta = solar_azimuth
12 alpha = solar_altitude
13
14 | # Calculate the area with sign correction
15 area = (np.\sin(np.\,radius(\,beta + room\_orientation)) *
              24) / np . tan ( np . radians ( alpha ) )
16
17 \parallel \parallel # Check if it is negative
18 if area < 0:
19 area = 0
20
21 # Cap the area at 40
22 if area > 40:
23 area = 40
24
25 return area
26
27 def calculate_total_sunlight_area (latitude, room_orientation
        ) :
\begin{array}{c|c}\n 28 & \text{total} \texttt{area} = 0 \\
 29 & \text{for day_of\_year}\n\end{array}for day_of_year in range (366): # Loop through all days
             of the year
30 for hour_of_day in range (25) : # Loop through all
                  hours of the day
31 total_area += calculate_sunlight_area ( latitude ,
                        day_of_year , hour_of_day , room_orientation )
32 return total_area
33
34 latitude = 23.0225 # Your latitude<br>35 gamma_range = range(361) # Room or
   35 gamma_range = range (361) # Room orientation from 0 to 360
        degrees
36
\frac{37}{38} total_areas = []<br>\frac{38}{10} for gamma in gam
\frac{38}{39} for gamma in gamma_range:<br>total area = calculat
         39 total_area = calculate_total_sunlight_area ( latitude ,
             gamma )
40 total_areas.append (total_area)
41
```

```
42 \parallel # Create a log plot<br>43 \parallel pH.semilogy (gamma
   plt . semilogy (gamma_range, total_areas)
44 plt.xlabel ('Room Orientation (gamma in degrees)')
_{45} \vert \rm\bar{p}lt. ylabel ('Total Sunlight Area (sq. ft)')
46 plt.title ('Total Sunlight Area vs. Room Orientation (Log
        Scale)')
47 | plt.grid (True)
48 | plt.show ()
```
7.3 Optimal Room Orientation Plot

Here is the source code for plot of the discrete sums vs room orienation. The code refers to the old library.py in section [7.1.1](#page-19-2) of the appendix containing the mathematical functions for calculations and a new plot.py using matplotlib to create the view.

7.3.1 optimize.py

```
import matplotlib. pyplot as plt
2 import numpy as np
3 from library import calculate_solar_altitude,
      calculate_solar_azimuth , calculate_declination ,
      calculate_hour_angle
4
5 def calculate_sunlight_area (latitude, day_of_year,
       hour_of_day , room_orientation ) :
6 declination = calculate_declination(day_of_year)
\frac{3}{7} hour_angle = calculate_hour_angle (hour_of_day)
8 \mid solar_altitude = calculate_solar_altitude (latitude,
           declination , hour_angle )
9 solar_azimuth = calculate_solar_azimuth ( hour_angle ,
           declination, 90 - solar_altitude)
10
11 \leftarrow beta = solar_azimuth<br>12 \leftarrow alpha = solar_altitu
       alpha = solar_altitude
13
14 # Calculate the area with sign correction
15 area = (np.\sin(np.\,radius(\,beta + room\_orientation)) *
           24) / np . tan ( np . radians ( alpha ) )
16
17 | # Check if it is negative
18 if area < 0:
19 area = 0
20
21 # Cap the area at 40
22 if area > 40:
23 area = 40
24
25 return area
26
27 def calculate_total_sunlight_area ( latitude , room_orientation
      ) :
28 total_area = 0
_{29} \mid \mid for day_of_year in range(366): # Loop through all days
           of the year
30 for hour_of_day in range (25) : # Loop through all
               hours of the day
31 total_area += calculate_sunlight_area (latitude,
                   day_of_year , hour_of_day , room_orientation )
```

```
32 return total_area
33
34 | latitude = 23.0225 # Your latitude
_{35} \mid gamma_range = range (361) \mid # Room orientation from 0 to 360
        degrees
36
37 | total_areas = []
38 for gamma in gamma_range:
39 total_area = calculate_total_sunlight_area (latitude,
            gamma )
40 total_areas.append (total_area)
41
42 # Find the minimal point<br>43 min_index = np.argmin(to
43 min_index = np.argmin (total_areas)<br>44 min_orientation = gamma_range [min_
44 min_orientation = gamma_range [ min_index ]
45 \texttt{min_value} = \texttt{total_areas} [\texttt{min\_index}]
\frac{46}{47}47 \neq Create a log plot<br>48 \mid plt.semilogy (gamma_
48 plt . semilogy ( gamma_range , total_areas )
49 plt.xlabel ('Room Orientation (gamma in degrees)')
_{50} \vert {\rm \bar{p}lt} . ylabel ('Total Sunlight Area (sq. ft)')
51 plt . title ( ' Total Sunlight Area vs . Room Orientation ( Log
         Scale ) ')
52 plt . grid ( True )
53
54 \parallel # Add a vertical line at the minimal point
_{55} |plt.axvline(x=min_orientation, color=^{\overline{\jmath}}red', linestyle='--',
         label = f ' Minimal Orientation : { min_orientation } degrees ')
56 plt . legend ()
57
58 | plt.show ()
```